Synthesis of State Estimation and $H_\infty$ Predictive Control for Networked Control Systems in automotive applications

Yiming ZHANG, Vincent SIRCOULOMB, Nicolas LANGLOIS

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Outline

- Introduction

- Robust estimation for networked control systems:
  - Application to battery state of charge estimation through a CAN bus

- $H_{\infty}$ predictive control for networked control systems:
  - Application to electromagnetic valve control through a CAN bus

- Conclusions and prospects
Industrial background

- Connect cyberspace to physical space
- Economical investments (unnecessary wiring)
- Data sharing, exchange

• Battery Management Unit (Battery state-of-charge estimation)
• Engine Management Unit (electromagnetic valve control)
Overview of main issues

State constraints
- Process (State)
- Sensors
- Transmitted Measurements
- Received measurements
- Dynamic model of the process
- Parameter uncertainties
- Process noise

Measurement

Network

Actuators

State estimator
- Estimated state
- Controller
- Desired output

Control signal
Network-induced issues

Model of a networked system

Process | Sensors
\[ y_k \]

\[ (k-3)T_s : \ y_{k-3} \]

\[ (k-2)T_s : \ y_{k-2} \]

\[ (k-1)T_s : \ y_{k-1} \]

\[ kT_s : \ y_k \]

- Network-induced delay
- Packet disordering
- Packet dropouts

Buffer

Estimator

\[ (k-3)T_s \]

\[ (k-2)T_s \]

\[ (k-1)T_s \]

\[ kT_s \]
Modeling of network characteristics

Measurement sequence over networks:

Full measurement sequence:

\[ Y_k = \{ y_1, \ldots, y_k \} \]

Bernoulli process:

\[ \gamma = \{ \gamma_1, \ldots, \gamma_k \}, \quad \gamma_i = 1 \text{ or } 0 \]

Incomplete and intermittent Measurements:

\[ Y_r \subseteq Y_k \]

\[ Y_r = \{ \gamma_1 y_1, \ldots, \gamma_k y_k \} \]

Packet arrival probability [Sinopoli, 2004]

\[ E[\gamma_k = 1] = \lambda \]
Robust state estimation for LTI system controlled over a network

Application to battery state of charge estimation
The estimator design

Dynamic model of the process:

\[
\begin{align*}
x_{k+1} &= \Phi x_k + \Gamma u_k + \omega_k \\
y_k &= c x_k + \nu_k
\end{align*}
\]

Kalman filter:

Prediction:

\[
\hat{x}_{k|k-1} = \Phi \hat{x}_{k-1} + \Gamma u_{k-1}
\]

\[
P_{k|k-1} = \Phi P_{k-1} \Phi^T + Q_c
\]

Update:

\[
\hat{x}_k = \hat{x}_{k|k-1} + \gamma_k K_k \left[ y_k - C \hat{x}_{k|k-1} - Du_k \right]
\]

\[
P_k = P_{k|k-1} C^T \left[ C P_{k|k-1} C^T + R \right]^{-1}
\]

Projection:

\[
\hat{x}_k = T \hat{x}_k + \Pi^{-1} F^T \left( F \Pi^{-1} F^T \right)^{-1} l
\]

when \( T = 1 \), it is the state-unconstrained system

The estimation error affected by:

- State constraints
- Varying process noise covariance
- Packet arrival probability
- Measurement noise covariance
A critical value [Zhang et al., 2013a, 2013b]

- Stability: Riccati equations with packet arrival probability

\[
\Omega\left(p^-,\lambda,Q\right) = \Phi T p^T T^T \Phi^T + Q - \lambda \Phi T p^T C^T \left[ C p^T C^T + R \right]^{-1} C p^T T^T \Phi^T
\]

\[
\Xi\left(p^+\lambda,Q\right) = T \Phi p^+ T^T T^T + T Q T^T - \lambda T \left( \Phi T p^+ T^T \Phi^T + Q \right) C^T \left[ C \left( \Phi T p^+ T^T \Phi^T + Q \right)^T C^T + R \right]^{-1} C \left( \Phi T p^+ T^T \Phi^T + Q \right) T^T
\]

- The existence of a critical packet arrival probability

we can find a critical value \( \lambda_c \) so that for \( \lambda \geq \lambda_c \)

Stability \( \forall 1 \geq \lambda \geq \lambda_c : \Omega\left(p^-,\lambda,Q\right) \leq \Omega\left(p^-,\lambda_c,Q\right) \leq p^- \quad \Xi\left(p^+\lambda,Q\right) \leq \Xi\left(p^+\lambda_c,Q\right) \leq p^+ \)

Unstability \( \forall 0 \leq \lambda < \lambda_c : \Omega\left(p^-,\lambda,Q\right) > \Omega\left(p^-,\lambda_c,Q\right) > p^- \quad \Xi\left(p^+\lambda,Q\right) > \Xi\left(p^+\lambda_c,Q\right) > p^+ \)

- The existence of upper bound of admitted process noise covariance

For \( Q_c \geq Q \geq Q_0 \),

\[
\begin{align*}
p^- & > \Omega\left(p^-,\lambda,Q_c\right) \geq \Omega\left(p^-,\lambda,Q\right) \geq \Omega\left(p^-,\lambda,Q_0\right) \\
p^+ & > \Xi\left(p^+\lambda,Q_c\right) \geq \Xi\left(p^+\lambda,Q\right) \geq \Xi\left(p^+\lambda,Q_0\right)
\end{align*}
\]

If \( \lambda_1 \leq \lambda_2 \),

\[
\Omega\left(p^-,\lambda_1,Q_c\right) \geq \Omega\left(p^-,\lambda_2,Q_c\right) \quad \Xi\left(p^+\lambda_1,Q_c\right) \geq \Xi\left(p^+\lambda_2,Q_c\right)
\]
• HEV motorization management management.
• What is the battery state-of-charge? (0%~100%)
• Internal variable in battery pack: SOC
Battery model and constraints

The battery electrical equivalent model [Banghu et al., 2005]:

\[
\begin{align*}
V(t) &= \frac{R_s}{R_s + R_e} V_C(t) + \frac{R_e}{R_s + R_e} V_e(t) + \left( \frac{R_s}{R_s + R_e} \right) i(t) \\
\frac{dV_{cb}(t)}{dt} &= \frac{-1}{C_b(R_e + R_s)} V_{cb}(t) + \frac{1}{C_s(R_e + R_s)} V_{cs}(t) + \frac{1}{C_b(R_e + R_s)} \frac{R_s}{R_s + R_e} \frac{R_e}{R_s + R_e} i(t) \\
\frac{dV_{cs}(t)}{dt} &= \frac{1}{C_s(R_e + R_s)} V_{cs}(t) + \frac{1}{C_b(R_e + R_s)} \frac{R_s}{R_s + R_e} \frac{R_e}{R_s + R_e} i(t)
\end{align*}
\]

The relationship between the bulk voltage and the SOC:

\[ V_{cb} = a \cdot SOC + b + n \]

Estimating the state of the battery model

Known parameters

Uncertainties

White noise
Input current and output voltage: experimental data

- Low and limited input current
Algorithm results for the unconstrained-state system

The SOC estimation problem:
• the nominal value of process noise covariance $Q_0$ ($Q_0 = 0.005*I$)

$$
\Phi = \begin{bmatrix}
0.9997 & 3.19e-4 \\
0.4126 & 0.5874
\end{bmatrix},
$$

$$
\Gamma = 
\begin{bmatrix}
2.1662e-6 \\
0.0018
\end{bmatrix},
$$

$$
C = [0.5 \ 0.5], \ D = 0.0041
$$
Algorithm results for the unconstrained-state system

- the upper value of uncertainty covariance \( Q_u = 0.01^* I \) when \( \lambda = 1 \)

\[
\begin{align*}
\text{A priori form} & : \lambda_{pri} = 0.5954, \quad \text{trace}(Q_{pri}) = 0.0117 \\
\text{A posteriori form} & : \lambda_{post} = 0.7533, \quad \text{trace}(Q_{post}) = 0.0119
\end{align*}
\]
Estimation performance for the unconstrained-state system

Comparison of mean absolute estimation Errors

<table>
<thead>
<tr>
<th></th>
<th>( \lambda_1 = 0.6016 )</th>
<th>( \lambda_2 = 0.1851 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Ave}_{pri} )</td>
<td>0.0934</td>
<td>0.9761</td>
</tr>
<tr>
<td>( \text{Ave}_{post} )</td>
<td>0.0049</td>
<td>0.8876</td>
</tr>
</tbody>
</table>
The augmented model with state constraints

Coulomb counting equation [Ng et al., 2009]:

\[
SOC_{k+1} = \frac{\eta T_s}{3600 C_{Ah}} u_k + SOC_k
\]

Augmented system model:

\[
x_{k+1} = \begin{bmatrix} V_{cb, k+1} \\ V_{cs, k+1} \\ SOC_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi & 0 \\ 0 & I \end{bmatrix} x_k + \begin{bmatrix} \Gamma \\ \frac{\eta T_s}{3600 C_{Ah}} \end{bmatrix} u_k + \begin{bmatrix} \omega_k \\ \omega_k^s \end{bmatrix}
\]

\[
y_k = \begin{bmatrix} C & 0 \end{bmatrix} x_k + D u_k + \nu_k
\]

Linear hard equality constraint equation:

\[
V_{Cb} - a \cdot SOC = b
\]
The robust results for the constrained-state system

The SOC estimation problem:
• the nominal value of process noise covariance $Q_0$ ($Q_0 = 0.005 \times I_3$)

<table>
<thead>
<tr>
<th></th>
<th>A priori form (Point A)</th>
<th>A posteriori form (Point B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{pri}$</td>
<td>0.505</td>
<td>0.6561</td>
</tr>
<tr>
<td>$\text{trace}(Q_{pri})$</td>
<td>0.0213</td>
<td>0.0212</td>
</tr>
</tbody>
</table>

Comparison of mean absolute estimation errors

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_1 = 0.55$</th>
<th>$\lambda_2 = 0.5$</th>
<th>$\lambda_3 = 0.1851$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave$_{pri}$</td>
<td>0.0682</td>
<td>0.0749</td>
<td>0.2042</td>
</tr>
<tr>
<td>Ave$_{post}$</td>
<td>0.0472</td>
<td>0.0682</td>
<td>0.1969</td>
</tr>
</tbody>
</table>
Comparison of constrained and unconstrained approaches

The a priori model form:

The a posteriori model form:

Comparison of mean absolute estimation errors

<table>
<thead>
<tr>
<th>λ = 0.6016</th>
<th>The state unconstrained approach</th>
<th>The state constrained approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave&lt;sub&gt;pri&lt;/sub&gt;</td>
<td>0.0934</td>
<td>0.013</td>
</tr>
<tr>
<td>Ave&lt;sub&gt;post&lt;/sub&gt;</td>
<td>0.0049</td>
<td>0.0045</td>
</tr>
</tbody>
</table>
From robust estimation to robust control
Control for LTI system controlled over a network

Application to electromagnetic valve control
Design of a predictive controller

• The predicted output sequence:

\[ \hat{Y}_k = \begin{bmatrix} \hat{y}_{k+m+1|k}^T, \hat{y}_{k+m+2|k}^T, \ldots, \hat{y}_{k+N|k}^T \end{bmatrix}^T \]

• The cost function:

\[
J(k, \Delta U_k) = \text{Exp} \left\{ \sum_{j=m+1}^{N} (y_{k+j|k} - \gamma_{k+j})^2 + \sum_{j=1}^{N_u} \mu_j (\Delta u_{k+j-1|k})^2 \right\}
\]

• The future reference sequence:

\[ R_k = \begin{bmatrix} \gamma_{k+m+1}^T, \gamma_{k+m+2}^T, \ldots, \gamma_{k+N}^T \end{bmatrix}^T \]

• The predicted control sequence:

\[ \Delta U_k = \begin{bmatrix} \Delta u_{k|k}^T, \Delta u_{k+1|k}^T, \ldots, \Delta u_{k+N_u-1|k}^T \end{bmatrix}^T \]

• Vector form of the cost function:

\[
J(k, \Delta U_k) = \begin{bmatrix} \hat{Y}_k - R_k \end{bmatrix}^T \begin{bmatrix} \hat{Y}_k - R_k \end{bmatrix} + \Delta U_k^T \mu \Delta U_k
\]
Considered state-space model

The state-space model:
\[
\begin{align*}
x_{k+1} &= \Phi x_k + \Gamma u_{k-m} + E \omega_k \\
y_k &= z_k + \nu_k = C x_k + \nu_k
\end{align*}
\]

Incremental state-space model:
\[
\begin{align*}
x_{k+1} &= (\Phi + I) x_k - \Phi x_{k-1} + \Gamma \Delta u_{k-m} + E \Delta \omega_k \\
y_k &= z_k + \nu_k = C x_k + \nu_k
\end{align*}
\]

• Proposal of a deterministic approach to compute incremental control signal in the case of parameter and process noise uncertainties
**H∞ design for uncertainties in parameter and process noise**

- **Control law computation from** \( \min_{\Delta U_k} J(N) \): 
  \[
  \Delta U_k = H (I-A_b) G^T \left[ A_a y_k - (I-A_b) M_1 \hat{x}_{k+m|k} + (I-A_b) M_2 \hat{x}_{k+m-1|k} \right]
  \]

- **The state-space model after introducing incremental control signal:** 
  \[
  x_{k+1} = \left[ \Phi + I - \Gamma \beta_1 H (I-A_b) G^T (I-A_b) M_1 \right] x_k - \left[ \Phi - \Gamma \beta_1 H (I-A_b) G^T (I-A_b) M_2 \right] x_{k-1} + E \Delta \omega_k
  \]

- **We define H∞ control performance index with a positive scalar \( r \) so that the system under the incremental control law should satisfy:** if (1) is satisfied for \( \Delta \omega_k \in \ell_2 [0, \infty) \), the system is H∞ norm stabilizable.
  \[
  \| Cx_k \|_2 \leq r \| \Delta \omega_k \|_2 \quad (1)
  \]

\[
\Lambda = \sum_{k=0}^{\infty} \left[ x_k^T C^T C x_k - r^2 \Delta \omega_k^T \Delta \omega_k \right] \leq 0
\]
**Proposed theorem [Zhang et al., 2013]**

For a given $r > 0$, if there are symmetric positive definite matrices $P$ and $Q$ so that the LMI is solvable, then the system is $H_\infty$ norm stable.

\[
\begin{bmatrix}
Q - P + C^T C & 0 & 0 & * \\
0 & -Q & 0 & * \\
0 & 0 & -r^2 I & * \\
\end{bmatrix} < 0
\]

where $X = P \Gamma \beta_1 H$, $K = P^{-1} X$

- **Computation of the control-weighting parameter:**

\[
\mu_1 = \frac{1}{\Gamma^T K K^T \Gamma} \left[ \Gamma^T \Gamma \beta_1 K^T \Gamma - \Gamma^T K (I - A_b) G^T (I - A_b) G K^T \Gamma \right]
\]

- **Parameter tuning:**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>......</th>
<th>$\alpha_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
<td>......</td>
<td>$\mu_n$</td>
</tr>
</tbody>
</table>
Electromagnetic valve

The kinetic model of the mechanical part of an solenoid valve:

\[ \theta \in [\theta_{\text{min}}, \theta_{\text{max}}] = [-4\text{mm}, +4\text{mm}] \]

The equivalent circuit of the electrical part of an solenoid valve:

- The coil current \( I_c \)
- The eddy current \( I_e \)
- The mass weight \( M \)
- The plunger

Electromagnetic valve: the mechanical and the electrical parts [Zhao, 2010].

What does the electromagnetic valve do?

The displacement of the valve:

- A valve lift (from -4mm to 4mm)
- A valve drop (from 4mm to -4mm)
Modelling and constraints

The linearized state-space valve model:

\[
\begin{bmatrix}
\delta \dot{\theta} \\
\delta \ddot{\theta} \\
\delta \dot{i}_c \\
\delta \dot{i}_e
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
\frac{\delta k_{\theta_{\text{max}}} - k_{\kappa}
d_f(\kappa - \theta_{\text{max}})}{M(\kappa - \theta_{\text{max}})} & \frac{f}{M} & \frac{2\sqrt{\beta k_{\theta_{\text{max}}}}}{M(\kappa - \theta_{\text{max}})} & 0 \\
0 & -\sqrt{\frac{k_{\theta_{\text{max}}}}{\beta}} & -R(\kappa - \theta_{\text{max}}) & \frac{R(\kappa - \theta_{\text{max}})}{2\beta} \\
0 & 0 & \frac{R}{L_e} & \frac{R + R_e}{L_e}
\end{bmatrix}
\begin{bmatrix}
\delta \theta \\
\delta \dot{\theta} \\
\delta i_c \\
\delta i_e
\end{bmatrix}

\delta u + \frac{\kappa - \theta_{\text{max}}}{1} + \frac{1}{M} \omega
\]

- **State:**
  - the displacement of the mass
  - the velocity of the mass
  - the eddy current

- **Uncertainties:**
  - process noise parameter
  - the relation between the combustion pressure and the valve position

**Outline**

- Introduction
- Battery SOC estimation
- Electromagnetic valve control
- Con.&Pros.

**Outline**

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# Model Parameters of the EM-driven Valve

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>M</td>
<td>0.194</td>
<td>kg</td>
</tr>
<tr>
<td>Coil resistance</td>
<td>R</td>
<td>0.52</td>
<td>Ω</td>
</tr>
<tr>
<td>Eddy current resistance</td>
<td>( R_s )</td>
<td>5</td>
<td>Ω</td>
</tr>
<tr>
<td>Eddy current inductance</td>
<td>( L_s )</td>
<td>5</td>
<td>mH</td>
</tr>
<tr>
<td>Displacement coefficient</td>
<td>( \kappa )</td>
<td>0.004025</td>
<td>m</td>
</tr>
<tr>
<td>Spring coefficient</td>
<td>( k_{sp} )</td>
<td>180</td>
<td>kN/m</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>( f )</td>
<td>12.75</td>
<td>kg/s</td>
</tr>
<tr>
<td>Positive maximal valve position</td>
<td>( \theta_{\text{max}} )</td>
<td>0.004</td>
<td>m</td>
</tr>
<tr>
<td>Maximal gas pressure to the valve</td>
<td>( P_{0\text{max}} )</td>
<td>100</td>
<td>bar</td>
</tr>
<tr>
<td>Valve diameter</td>
<td>( D )</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Slope of linearized relation between pressure and valve position</td>
<td>( d_f )</td>
<td>-625</td>
<td>kN/m</td>
</tr>
<tr>
<td>Coil current coefficient</td>
<td>( \beta )</td>
<td>( 10^{-5} )</td>
<td>H · m · turn</td>
</tr>
</tbody>
</table>
Electromagnetic valve control system

- Desired Torque from the Driver
- Engine Management Unit
  - Measurements (pressure, temperature...)
  - Valve Position Reference
- Electronic Control Unit
  - Control signal $u_{k+m|k}$
  - Valve Position Feedback
- Engine ECU
- References for other local control units
- CAN Bus
  - Control signal
  - m size buffer
- Sensor
- Valve
- Outline
  - Introduction
  - Battery SOC estimation
  - Electromagnetic valve control
  - Con.&Pros.

Mathematical equations:
\[
\theta^r_{k+m} = \theta_{k+m}|_k + ku
\]
Reference setting

Driver-desired reference of valve position

- A complete valve motion
  Discretization: 10 microseconds
- Time interval determines the engine speed (from 18000rpm to 12000rpm)
  18000rpm: 0.00667s
  12000rpm: 0.01s

Valve lift

Valve drop

Time interval between two complete valve motions
Predictive reference strategy and controller setting

![Graph showing valve position reference and simulation time](image)

- The Driver-Desired Reference
- The Predictive Reference

### Parameters setting for GPC

- $5.5 \times 10^{-2}$
- $2 \times 10^{-2}$
- $0.78 \times 10^{-2}$
- $8.7 \times 10^{-2}$
- $3 \times 10^{-2}$
- $1 \times 10^{-2}$
- $10 \times 10^{-2}$
- $5.45 \times 10^{-2}$
- $2.45 \times 10^{-2}$

With $5 \, \mu m$

With $10 \, \mu m$

With $20 \, \mu m$

### Outline

- Introduction
- Battery SOC estimation
- Electromagnetic valve control
- Con. & Pros.
System output, error and valve velocity

Zoom-in between 103.5ms and 106ms

Valve position error for the whole simulation.

System output, error and valve velocity

Zoom-in between 103.5ms and 106ms

Valve position

Valve position error

Valve velocity

for the whole simulation.

COV

$\alpha = 0.2$  
$\alpha = 0.5$  
$\alpha = 0.7$

$m = 5$  
$0.014026e^{-4}$  
$0.017219e^{-4}$  
$0.018528e^{-4}$

$m = 10$  
$2.3731e^{-4}$  
$3.1546e^{-4}$  
$3.8832e^{-4}$

$m = 20$  
$563e^{-4}$  
$572e^{-4}$  
$577e^{-4}$
Features: valve motion and control response

The valve opening/landing velocity (18000rpm)

<table>
<thead>
<tr>
<th>Valve Position θ (m)</th>
<th>α = 0.2</th>
<th>α = 0.5</th>
<th>α = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>landing</td>
<td>opening</td>
<td>landing</td>
</tr>
<tr>
<td>0.0205</td>
<td>-0.0205</td>
<td>0.0171</td>
<td>-0.0171</td>
</tr>
<tr>
<td>0.5125%</td>
<td>0.5125%</td>
<td>0.4275%</td>
<td>0.4275%</td>
</tr>
<tr>
<td>0.02</td>
<td>-0.02</td>
<td>0.0193</td>
<td>-0.0193</td>
</tr>
<tr>
<td>0.5%</td>
<td>0.5%</td>
<td>0.4825%</td>
<td>0.4825%</td>
</tr>
<tr>
<td>0.0201</td>
<td>-0.0201</td>
<td>0.0184</td>
<td>-0.0184</td>
</tr>
<tr>
<td>0.5025%</td>
<td>0.5025%</td>
<td>0.46%</td>
<td>0.46%</td>
</tr>
</tbody>
</table>

ESIGELEC
### Features:

- Valve motion and control response

---

**Table 1: Valve Opening/Landing Velocities**

<table>
<thead>
<tr>
<th>$m = 5$</th>
<th>$\alpha = 0.2$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a$ (m/s)</td>
<td>7.4674</td>
<td>8.4944</td>
<td>8.8851</td>
</tr>
<tr>
<td>$V_b$ (m/s)</td>
<td>4.1488</td>
<td>3.426</td>
<td>3.1824</td>
</tr>
<tr>
<td>$V_c$ (m/s)</td>
<td>-7.6841</td>
<td>-8.6696</td>
<td>-8.8915</td>
</tr>
<tr>
<td>$V_d$ (m/s)</td>
<td>-4.1463</td>
<td>-3.4305</td>
<td>-3.1918</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>$\alpha = 0.2$</th>
<th>$m = 5$</th>
<th>$m = 10$</th>
<th>$m = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a$ (m/s)</td>
<td>7.4674</td>
<td>6.7162</td>
<td>6.5229</td>
</tr>
<tr>
<td>$V_b$ (m/s)</td>
<td>4.1488</td>
<td>4.7286</td>
<td>4.8937</td>
</tr>
<tr>
<td>$V_c$ (m/s)</td>
<td>-7.6841</td>
<td>-6.7853</td>
<td>-6.5421</td>
</tr>
<tr>
<td>$V_d$ (m/s)</td>
<td>-4.1463</td>
<td>-4.7363</td>
<td>-4.9001</td>
</tr>
</tbody>
</table>

- A large starting valve velocity ($V_a$ and $V_c$)
- A small arrival valve velocity ($V_b$ and $V_d$)
Conclusion and prospects

Conclusion:

1) Design of a network-faced state-constrained estimator robust to varying uncertainties in both parameters and process noise
   • Application to battery state-of-charge estimation problem

2) Proposal of a deterministic control law from a $H_\infty$ predictive controller
   • Application to electromagnetic valve control problem

Prospects:

1) Relax the strong assumption for noise
2) Consider nonlinear systems
3) Consider a variable sampling period method
THANKS FOR YOUR ATTENTION