Abstract: This paper investigates H\(\infty\) predictive control problem for a class of networked control system, and presents a method for quantitatively selecting the control weighting parameter. Based on Lyapunov function approach and H\(\infty\) control theory, a generalized predictive state feedback controller is derived to guarantee the closed-loop networked system stable. Then, an algebraic method for selecting a feasible GPC control-weighting parameter is proposed. The contribution realizes the adaptive parameter tuning for GPC with respect to response demands and performance. A simulation example demonstrates the feasibility and practicability of the proposed method.

Keywords: Lag Networks, Generalized Predictive Control, Adaptive control, H-infinity control, Parameter Optimization

1. INTRODUCTION

Networked Control Systems (NCS) (Wang et al., 2008, and Hespanha et al., 2007) has been widely applied to many areas such as vehicle control or industrial process, which can realize information sharing and communication among all the devices of a system. Meanwhile, along with its advantages, network also induces new problems of time delays, packet losses, and out-of-order (Luo and Chen, 2000), which degrade the control performance of the controlled system and may even cause the system to be unstable (Zhang et al., 2001).

As we know, network-induced delays are stochastic, disordering, unpredictable, and confusing. To overcome unknown delay and dropout, generalized predictive control (GPC) (Clarke et al., 1987) begins to be extended to networked systems. Its control prediction generator is designed to generate a set of future control laws in order to cover the situation of no arrival packet (Hu, Bai, Shi and Wu, 2007, Lam, Gao and Wang, 2007, and Xiong and Lam, 2007). However, ordinary GPC lacks of stability analysis and its parameters such as smooth one or control-weighting one are always designed by experience. Furthermore, few papers referring to GPC talk about the topic of disturbance problem.

Motivated by the above mentioned problem, in this paper, a newly model of generalized predictive control is proposed, and based on this model, a quantized H\(\infty\) problem for networked systems is studied. The first contribution is to solve the stability problem for generalized predictive control, and also achieve the prescribed H\(\infty\) disturbance level. The second contribution is to provide a method of quantitative selection for the control-weighting parameter. Based on two above results, we can get a relation table between the smooth parameter and the control-weighting parameter. We assume that: when the plant is running, the predictive controller determines a proper smooth parameter by following a rule (For example, estimation through a least square algorithm). Then, it achieves an adaptive control-weighting parameter guaranteeing the system stable and better performance. A numerical example is given to illustrate the effectiveness of the proposed GPC design method.

This paper is organized as follows. Section 2 gives an introduction to the modelling of the system. Modelling of network-faced predictive control under bounded H\(\infty\) norm is presented in Section 3. The proposed method is experimented in Section 4 and concluding remarks are given in Section 5.

2. PROBLEM STATEMENT

Let us consider a noisy discrete-time model obtained from the discretization of a continuous time-invariant plant with sampling time \(T_s\). For such a networked control system, the state-space model of the physical discrete-time system is assumed to satisfy:

\[
\begin{align*}
    x_{k+1} &= \Phi x_k + \Gamma u_k + E_0 k
    
    z_k &= C x_k
    
    y_k &= z_k + u_k
\end{align*}
\]

where at time \(k T_s\), \(x_k \in \mathbb{R}^n\) is the state vector, \(z_k \in \mathbb{R}^m\) is the output vector, \(y_k \in \mathbb{R}^p\) is the sensor’s feedback, \(u_k \in \mathbb{R}^p\) is the control vector, and \(\Phi, \Gamma, C\) are constant matrices of appropriate dimension. \(\{\omega_k\}\) and \(\{\nu_k\}\) are Gaussian, uncorrelated, white, and zero mean processes.

Hereby, we consider a class of NCS with short network-induced delays in the forward channel (Fig. 1). Timestamps (Nilsson, 1998, and Nilsson et al., 1998) keep the same clock. Any out-of-order arrival of packets can be reordered within one buffer span.

The model (1) is equivalent to the following incremental state-space model:
The topic about how to compute the expectations on the states at time \( k \), is the future reference trajectory, \( N \) is the maximal predictive horizon, \( N_1 \) is the minimal predictive horizon (1 for this situation, \( n \geq 1 \), and \( N - N_1 \geq N_u \)), \( N_u \) is the control horizon and \( \lambda_j \) is the weighting sequence, and where \( y_{k+j} \) is the j-step future output computed as follows:

\[
y_{k+j} = \sum_{i=0}^{N_j} C_i y_{i-1} + \sum_{j=0}^{b} \sum_{h=0}^{b} C_{h,j} y_{k+j} + \sum_{i=0}^{N_j} \lambda_j (\Delta u_{k+j-i})
\]

The optimal predictor of \( y_{k+j} \) is:

\[
\hat{y}_{k+j} = \mathbb{E}[y_{k+j}] = \sum_{i=0}^{N_j} C_i \mathbb{E}[x_{i-1}] + \sum_{j=0}^{b} \sum_{h=0}^{b} C_{h,j} \mathbb{E}[x_{k+j}]
\]

The topic about how to compute the expectations \( \mathbb{E}[x_k] \) and \( \mathbb{E}[x_{k+1}] \) have been discussed (Zhang et al., 2013).

Let us define:

\[
\hat{y}_k = \left[ \hat{y}_{1+k}^T, \hat{y}_{2+k}^T, \ldots, \hat{y}_{N+k}^T \right]^T
\]

\[
\Delta U_k = \left[ \Delta u_{k+1}^T, \Delta u_{k+2}^T, \ldots, \Delta u_{N+k}^T, \Delta u_{N+k-1}^T \right]^T
\]

Hence, the optimal future outputs can be simplified by the vector form:

\[
\hat{y}_k = G\Delta U_k + T_k
\]

Defining

\[
\lambda = \text{diag} \left\{ \lambda_1, \lambda_2, \ldots, \lambda_{N_1} \right\},
\]

\[
R_k = \left[ y_{k+2}^T, y_{k+3}^T, \ldots, y_{k+N}^T \right]^T,
\]

\[
Y_k = \left[ y_{k+2}, y_{k+3}, \ldots, y_{k+N} \right]^T
\]

We get the vector form of the cost function:

\[
J(k, \Delta U_k) = \mathbb{E} \left[ \left[ Y_k - R_k \right] \left[ Y_k - R_k \right]^T + \Delta U_k^T \lambda \Delta U_k \right]
\]

\[
= \left[ \hat{y}_k - R_k \right]^T \left[ \hat{y}_k - R_k \right] + \Delta U_k^T \lambda \Delta U_k
\]

Taking into account packet dropouts and delay in the feedback channel, the future reference trajectory is specially designed by the expectation of output, which is given by the following rule:

\[
y_{k+2} = \alpha y_{k+1} + (1-\alpha) y_k
\]

\[
y_{k+j} = \alpha^j y_{k+j} + (1-\alpha^j) y_k\), \( j \geq 2
\]

where \( \alpha \) is the smooth parameter, \( 0 \leq \alpha \leq 1 \).

So,

\[
R_k = \begin{bmatrix}
0 & \ldots & 0 \\
0 & \alpha^2 & \ldots & 0 \\
0 & \ldots & \alpha^{N-1} & 0 \\
0 & \ldots & 0 & \alpha^{N-1}
\end{bmatrix}
\]

\[
\hat{y}_k = A_{\Delta k} \hat{y}_k + A_{\alpha} \gamma_k
\]

By minimizing the cost function following

\[
\partial J(k, \Delta U_k) / \partial \Delta U_k = 0,
\]

then

\[
\Delta U_k = \left[ (I - A_{\Delta}) G^T (I - A_{\Delta}) G + \lambda \right]^{-1} (I - A_{\Delta}) G^T A_{\alpha} \gamma_k
\]

\[
- (I - A_{\Delta}) M_2 \hat{y}_{k+1} + (I - A_{\Delta}) M_2 \hat{y}_k
\]
For the sake of simplicity, let us introduce the symbol $\beta_i$ for denoting every row of $H = (I-A_b)G^T(I-A_b)G+\lambda I)^{-1}$:

$$
\beta_i = \begin{bmatrix} 0_{n_i} & 1 & 0_{n(N_u-i-1)} \end{bmatrix}
$$

where $0_{n \times m}$ is the null matrix with $n$ lines and $m$ columns.

The component in the control horizon computed at time $(k-d)T_e$ is then:

$$
\Delta u_{k-d} = \beta_d H (I-A_b) G^T \left( A_{d} y_k - d (I-A_b) M_2 \hat{z}_{k+d} \right)
+ (I-A_b) M_2 \hat{z}_{k-d}
$$

where $1 \leq d \leq N_u - 1$.

According to this control law, in fact, generalized predictive control is equivalent to state feedback control. The interest of this paper is to stabilize the system (1) by a proper gain. The functional relationship between the gain and the control-weighting parameter $\lambda$ provides a method of quantitative selection.

For the time-delay system (2), we design a one-step ahead predictive control law as $H_\infty$ controller, which should satisfy:

Given a constant positive $r$ and zero initial condition, if $\|z_k\|_\infty < \|\Delta \omega_k\|_\infty$, is satisfied for $\Delta \omega_k \in L^2(0,\infty)$, the closed-loop system (2) is $H_\infty$ norm stabilizable.

Lemma 1 (The Schur complement lemma (Zhang, 2005, and Haynsworth, 1968)): Given constant matrices $s_1$, $s_2$, $s_3$ of appropriate dimensions where $s_1$ and $s_2$ are symmetric, then $s_2 > 0$ and $s_1 + s_2^T S_3 < 0$ if and only if

$$
\begin{bmatrix}
 s_1 & s_2^T \\
 s_3 & -s_2
\end{bmatrix} < 0,
$$
or equivalently

$$
\begin{bmatrix}
 -s_2 & s_3 \\
 s_3^T & s_1
\end{bmatrix} < 0.
$$

Theorem 1: For a given $r > 0$, if there exist symmetric positive definite matrices $P$ and $Q$ such that LMI (11) is solvable, then the closed-loop system (2) is $H_\infty$ norm stabilizable.

$$
\begin{bmatrix}
 Q - P C^T C & 0 & 0 & 0 \\
 0 & -Q & 0 & 0 \\
 0 & 0 & -r^2 & 0 \\
 R \Phi \gamma P X (I-A_b) G^T (I-A_b) M_2 - P \Phi & E & -P
\end{bmatrix} \preceq 0
$$

where $X = P \gamma H$ and the stars means its corresponding transpose matrix:

$$
\begin{bmatrix}
 Z^T \\
 Z
\end{bmatrix} \preceq \begin{bmatrix} * \\
 * \end{bmatrix}
$$

Proof: substituting the first control signal at time $(k-1)T_e$ into the system and considering $\hat{x}_k = x_k - \hat{x}_k$, $x_{k+1} = (\Phi + I) x_k - \Phi \alpha_{k-1} + \Gamma \beta H (I-A_b) G^T (A_d y_k + I-A_b) M_1 \hat{x}_k + E \Delta \omega_k$

$$
= (\Phi + I - \Gamma \beta H (I-A_b) G^T (I-A_b) M_1) x_k
- (\Phi - \Delta \omega_k H (I-A_b) G^T (I-A_b) M_2) x_{k-1} + E \Delta \omega_k + \Theta \hat{z}_k
$$

where $\Theta = \Gamma \beta H (I-A_b) G^T (I-A_b) M_1 - \Gamma \beta H (I-A_b) G^T (I-A_b) M_2$.

$\Gamma \beta H (I-A_b) G^T A_d$

Hereby, the separation principle is guaranteed to be applicable and allows for reducing the study of the closed-loop dynamic to a simplified stabilization problem.

$$
x_{k+1} = (\Phi + I - \Gamma \beta H (I-A_b) G^T (I-A_b) M_1) x_k
- (\Phi - \Delta \omega_k H (I-A_b) G^T (I-A_b) M_2) x_{k-1} + E \Delta \omega_k
$$

We assume that there exist positive symmetric matrices $P$ and $Q$. Then, we can choose the following Lyapunov function:

$$
V_k = x_k^T P x_k + \sum_{i=k-d}^{k-1} x_i^T Q x_i
$$

$$
= x_k^T P x_k + x_i^T Q x_i
$$

Its forward differential equation is given by:

$$
\Delta V_k = V_{k+1} - V_k = x_{k+1}^T P x_{k+1} + x_{k+1}^T Q x_{k+1} - x_k^T P x_k - x_{k-1}^T Q x_{k-1}
$$

$$
= x_{k+1}^T P x_{k+1} + x_{k}^T (Q - P) x_k - x_{k-1}^T Q x_{k-1}
$$

$$
= \begin{bmatrix}
 x_k \\
 x_{k-1} \\
 \Delta \omega_k
\end{bmatrix}^T \begin{bmatrix}
 \Phi + I - \Gamma \beta H (I-A_b) G^T (I-A_b) M_1 \\
 \Gamma \beta H (I-A_b) G^T (I-A_b) M_2 - \Phi \\
 E^T
\end{bmatrix} P \begin{bmatrix}
 x_k \\
 x_{k-1} \\
 \Delta \omega_k
\end{bmatrix}
$$

where for any matrix $Z$: $ZP \begin{bmatrix} x_k \\
 x_{k-1} \\
 \Delta \omega_k
\end{bmatrix} = ZP \begin{bmatrix} x_k \\
 x_{k-1} \\
 \Delta \omega_k
\end{bmatrix}$.

The $H_\infty$ cost function $H$ is:

$$
H = \sum_{k=0}^{\infty} \sum_{k=0}^{\infty} \left[ z_k^T z_k - r^2 \Delta \omega_k^2 \Delta \omega_k \right] \leq \sum_{k=0}^{\infty} \left[ z_k^T z_k - r^2 \Delta \omega_k^2 \Delta \omega_k \right] + V_k
$$

$$
= \sum_{k=0}^{\infty} \left[ z_k^T z_k - r^2 \Delta \omega_k^2 \Delta \omega_k + \Delta V_k \right]
$$
where

\[ \sum_{k=0}^{\infty} \begin{bmatrix} x_k \\ x_{k-1}/\Delta\omega_k \end{bmatrix}^T \begin{bmatrix} \Phi + I - \Gamma \beta H(I-A_h)G^T(I-A_h)M_1 \\ \Gamma \beta H(I-A_h)G^T(I-A_h)M_2 - \Phi \end{bmatrix}^T P^T \begin{bmatrix} Q - P + C^T C \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1}/\Delta\omega_k \end{bmatrix} = 0 \].

According to lemma 1,

\[ \begin{bmatrix} \Phi + I - \Gamma \beta H(I-A_h)G^T(I-A_h)M_1 \\ \Gamma \beta H(I-A_h)G^T(I-A_h)M_2 - \Phi \end{bmatrix}^T P^T \begin{bmatrix} Q - P + C^T C \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -Q^T - Q^T \\ 0 \end{bmatrix} \leq 0 \]

\[ \begin{bmatrix} \Phi + P - \Gamma \beta H(I-A_h)G^T(I-A_h)M_1 \\ \Gamma \beta H(I-A_h)G^T(I-A_h)M_2 - \Phi \end{bmatrix}^T P^T \begin{bmatrix} Q - P + C^T C \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -Q^T - Q^T \\ 0 \end{bmatrix} \leq 0 \]

\[ \begin{bmatrix} Q - P + C^T C \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -Q^T - Q^T \\ 0 \end{bmatrix} \leq 0 \]

\[ \begin{bmatrix} P \Phi + P - \Gamma \beta H(I-A_h)G^T(I-A_h)M_1 \\ \Gamma \beta H(I-A_h)G^T(I-A_h)M_2 - \Phi \end{bmatrix} \begin{bmatrix} Q - P + C^T C \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -Q^T - Q^T \\ 0 \end{bmatrix} \leq 0 \]

where \( X = \Gamma \beta \). If this LMI is solvable by a bounded \( r \), the networked control system is \( H_\infty \) norm stabilizable. \( \lambda \) can be computed by:

\[ \Gamma \beta H = P^{-1} X = K \]

\[ \Rightarrow \Gamma \beta = K(I-A_h)G^T(I-A_h)K + \lambda I \]

\[ \Rightarrow \lambda = \frac{1}{\Gamma K K^T \Gamma} [\Gamma^T \beta^T K^T \Gamma - (I-A_h)G(I-A_h)G^T(I-A_h)K^T \Gamma] \]

Proof done.

4. SPECIAL CASES AND EXAMPLES

Consider a discrete linear time-invariant plant like (1) with sampling period \( T_s = 0.1 \), and:

\[ \Phi = \begin{bmatrix} 1.2 & -0.7 \\ 1 & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [0.5 \ 0.1], E = [0.1 \ 0.1]; \]

\[ \omega_N \sim N(0,Q) \text{ with variance } Q = 20 \cdot I_2; \]

\[ \nu_{kN} \sim N(0,R) \text{ with variance } R = 2.5. \]

where \( I_m \) denotes the identity matrix of dimension \( m \).

Considering \( r = 1 \), \( N_p = 3 \) and \( N_s = 4 \), this paper selects three smooth parameters for samples. By means of Matlab LMI toolbox, we compute corresponding feasible solutions based on theorem 1.

Table 1. Results of \( H_\infty \) norm-based parameter tuning

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.2</th>
<th>0.5</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.0043</td>
<td>0.0021</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Table 2. Three situations for parameter tuning

<table>
<thead>
<tr>
<th>Simulation period</th>
<th>0–10</th>
<th>10–15</th>
<th>15–25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( \alpha = 0.7 )</td>
<td>( \alpha = 0.7 )</td>
<td>( \alpha = 0.7 )</td>
</tr>
<tr>
<td>Case 2</td>
<td>( \alpha = 0.7 )</td>
<td>( \alpha = 0.5 )</td>
<td>( \alpha = 0.5 )</td>
</tr>
<tr>
<td>Case 3</td>
<td>( \alpha = 0.7 )</td>
<td>( \alpha = 0.5 )</td>
<td>( \alpha = 0.2 )</td>
</tr>
</tbody>
</table>

According to Tab. 2, we obtain the simulation results of control signals and outputs.
Figure 4 demonstrates that case 3 achieves the smallest difference. For three cases, it is clear that the presented self-adaptation approach can lower the system overshoot and smooth the system response. The computation and selection of parameters can be automatically finished online.

Meanwhile, a larger smooth parameter will lead to static error, which refers to case 1 (Fig. 3). As a result, we can find that the results keep the similar law with the rule from experience.

5. CONCLUSION

This paper provides a necessary and sufficient condition for the stabilization problem of generalized predictive controller under the bounded $H_\infty$ norm. Based on LMI, Lyapunov and $H_\infty$ control theory, a discrete LMI-based theorem has been presented and the control-weighting parameter can be quantified by a derived algebraic method. It realizes the adaptation control for GPC with the self-adjustable smooth parameter and the online computed control-weighting parameter, especially for the reference with sudden jumps.

As a result, simulation demonstrates that computed LMI-based feasible solutions can keep the system stable under $H_\infty$ norm and the environment of network-induced delay.

REFERENCES


